Math 111 Rational Functions Worksheet

For each of the following functions find the domain, y-intercept, x-intercept(s), holes, vertical asymptotes and horizontal asymptotes:

1. $f(x) = \dfrac{x+1}{x+2}$
2. $g(x) = \dfrac{3(x-2)(x-5)}{(x-2)(x+1)}$
3. $h(x) = \dfrac{1}{x^2-x-1}$
4. $k(x) = \dfrac{x^2-2x-1}{2x-2}$
5. $\ell(x) = \dfrac{2x-1}{2x^2-7x+3}$
6. $m(x) = \dfrac{-3(x^2+2x+1)}{4(x^2-1)}$
7. $s(x) = \dfrac{5x^2-20}{3x^2+12x+12}$

1. Give an example of a rational function p which satisfies all of the following properties:
   (a) $p(3) = 0$
   (b) $p(2)$ is not defined
   (c) p has a horizontal asymptote at $y = \frac{1}{3}$
2. Give an example of a rational function q which satisfies all of the following properties:
   (a) $q(8) = 0$
   (b) q has a hole at $x = 4$
   (c) q has a horizontal asymptote at $y = 0$
3. Give an example of a rational function R which satisfies all of the following properties:
   (a) R has a zero at $x = 2$
   (b) R has a hole at $x = 3$
   (c) R has a vertical asymptote at $x = 4$
   (d) R has a horizontal asymptote at $y = -\frac{2}{3}$
4. Give an example of a rational function which has no holes or vertical asymptotes and a horizontal asymptote at $y = 0$.

Answers

1. (a) y-intercept: $(0, \frac{1}{2})$
   (b) x-intercept(s): $(-1, 0)$
   (c) Hole(s): NONE
   (d) Vertical Asymptote(s): $x = -2$
   (e) Horizontal Asymptote: $y = 1$

2. (a) y-intercept: $(0, -15)$
   (b) x-intercept(s): $(5, 0)$
   (c) Hole(s): at $(2, -3)$
   (d) Vertical Asymptote(s): $x = -1$
   (e) Horizontal Asymptote: $y = 3$

3. (a) y-intercept: $(0, -1)$
   (b) x-intercept(s): NONE
   (c) Hole(s): NONE
   (d) Vertical Asymptote(s): $x = \dfrac{1+\sqrt{5}}{2}$ and $x = \dfrac{1-\sqrt{5}}{2}$
   (e) Horizontal Asymptote: $y = 0$

4. (a) y-intercept: $(0, \frac{1}{2})$
   (b) x-intercept(s): $(1+\sqrt{2}, 0)$ and $(1-\sqrt{2}, 0)$
   (c) Hole(s): NONE
   (d) Vertical Asymptote(s): $x = 1$
   (e) Horizontal Asymptote: NONE

5. (a) y-intercept: $(0, -\frac{1}{3})$
   (b) x-intercept(s): NONE
   (c) Hole(s): at $(\frac{1}{2}, -\frac{2}{5})$
   (d) Vertical Asymptote(s): $x = 3$
   (e) Horizontal Asymptote: $y = 0$

6. (a) y-intercept: $(0, \frac{3}{4})$
   (b) x-intercept(s): NONE
   (c) Hole(s): at $(-1, 0)$
   (d) Vertical Asymptote(s): $x = 1$
   (e) Horizontal Asymptote: $y = -\frac{3}{4}$

7. (a) y-intercept: $(0, -\frac{5}{3})$
   (b) x-intercept(s): $(2, 0)$
   (c) Hole(s): NONE
   (d) Vertical Asymptote(s): $x = -2$
   (e) Horizontal Asymptote: $y = \frac{5}{3}$

For each of the following there are many possible answers. I will only give one answer to each.

1. $p(x) = \dfrac{x-3}{3x-6}$

2. $q(x) = \dfrac{(x-8)(x-4)}{(x-4)(x^2+1)}$

3. $R(x) = \dfrac{-2(x-2)(x-3)}{3(x-3)(x-4)}$

4. The simplest example here is $f(x) = 0$, another example is $f(x) = \dfrac{1}{x^2+5}$.